Exercise 1

1. Calculate the lowest two energy levels $E_1$, $E_2$, and $E_{QM}=(E_2-E_1)$ for a quantum well of width $l$, as shown in Fig. 1.2.2, using the kinetic energy $E = p^2/2m$ and the de Broglie relation $p = h/\lambda$ between the momentum $p$ and the wavelength $\lambda$. How small does the width $l$ have to be in order to have a level spacing $E_{QM} = 0.025 \text{ eV}$ (which is the thermal energy $kT$ at room temperature)? Planck’s constant $h = 6.6 \times 10^{-34} \text{ Js}$, electron mass $m = 9.1 \times 10^{-31} \text{ kg}$, $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$, $J = \text{ kg m}^2/\text{s}^2$.

2. Calculate the Coulomb $E_C$ energy for putting a single electron onto a sphere of radius $r$ embedded in a dielectric with the dielectric constant of silicon ($\varepsilon=12$). Use the relation $E_C = qV$, where $q$ is the charge and $V$ the electrostatic potential (= voltage) of the charged sphere. Use $V = q/\varepsilon r$ (cgs units) or $V = q/4\pi\varepsilon_0\varepsilon r$ (SI units). How small does the diameter $d = 2r$ have to be in order to have a charging energy $E_C = 0.025 \text{ eV}$? The charge of an electron is $e = 4.8 \times 10^{-10} \text{ esu}$ (cgs units), $1 \text{ eV} = 1.6 \times 10^{-12} \text{ erg}$ (cgs units), $e^2/4\pi\varepsilon_0 = 1.44 \text{ eV nm}$ (SI units).